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The 12th Biennial Conference on Classical and Quantum Relativistic Dynamics of Particles and Fields

1 - 4 June 2020 ♦ Virtual Meeting Online

1 - 4 September 2020 ♦ Czech Technical University in Prague

June 2, 2020

Matthew A. Trump
Arizona, U.S.A.

The Darwin LAGRANGIAN at 100

COULOMB GAUGE

$$L_{\mathrm{D}} = \frac{q_1 q_2}{r} \frac{1}{2c^2} \mathbf{v}_1 \cdot [\mathbf{1} + \hat{\mathbf{r}}\hat{\mathbf{r}}] \cdot \mathbf{v}_2$$

"The dynamical motions of charged particles", C. G. Darwin, *Phil. Mag.* 6, 1920.

Two-body relativistic effects correct to order $(v/c)^2$

Helium-like atoms

Sir Charles Galton Darwin (1887-1962)

aka

A Brief History of Dynamical Time

A look back (and forward) at a century of the search for a manifestly covariant dynamics of many bodies with interaction

Breit Operator (1929)

$$\hat{H}_2 = -\sum_{i>j} \frac{q_i q_j}{2 r_{ij} m_i m_j c^2} \left[\hat{\mathbf{p}}_i \cdot \hat{\mathbf{p}}_j + \frac{(\mathbf{r}_{ij} \cdot \hat{\mathbf{p}}_i)(\mathbf{r}_{ij} \cdot \hat{\mathbf{p}}_j)}{r_{ij}^2} \right]$$

Lowest order perturbation of the Dirac equation

aka "SHP 101"

(Stueckelberg-Horwitz-Piron)

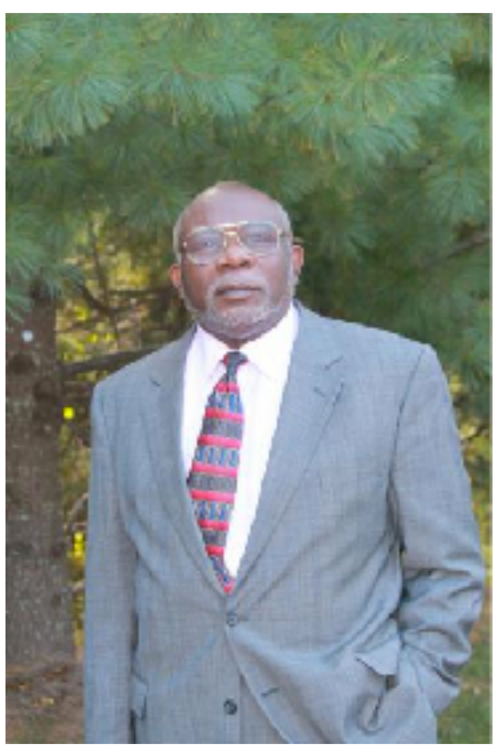
(Classical (mostly))

June 2, 2020

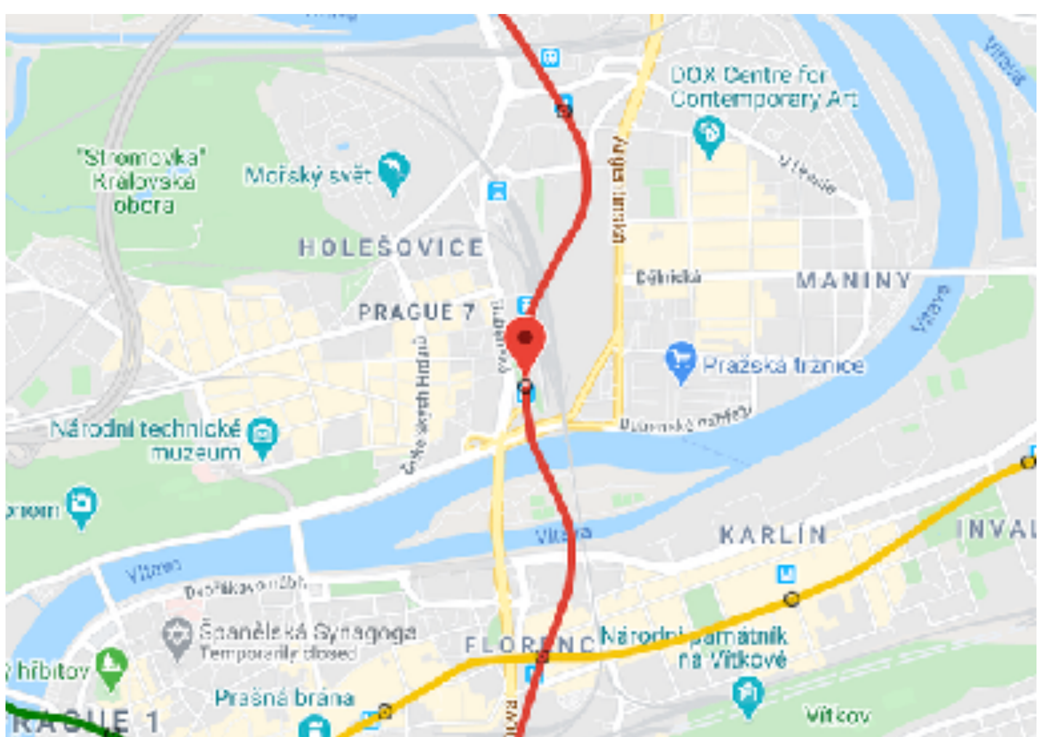
Matthew A. Trump

Arizona, U.S.A.

IARD
2020

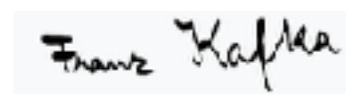


Prague Typical day c. 2020










PRAHA


Svatá prostota!

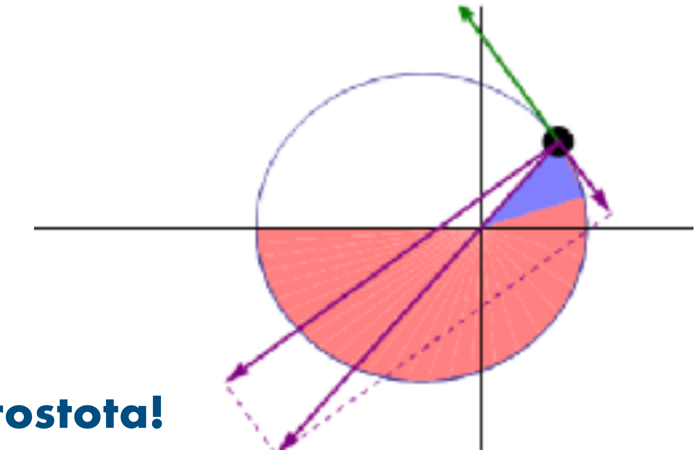

Kepler

1571-1630

Prague 1600-1612

1600-1601 as assistant to Tycho Brahe

Theory



Svatá prostota!

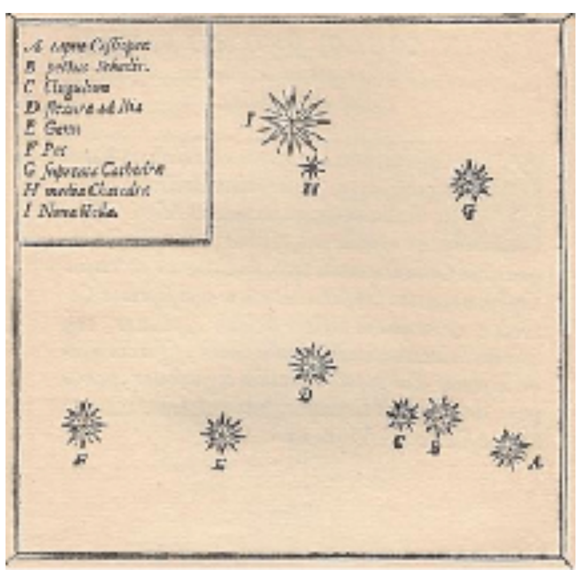
Tycho Brahe

1546 – 1601

born **Tyge Ottesen Brahe**

Scania (then Denmark, now Sweden)

1597 Brahe, age 51, is exiled from Denmark. He is invited to become the official imperial astronomer in Prague by Rudolf II, Emperor and King of Bohemia.



Observation

IARD

"to facilitate the acquisition and dissemination of knowledge about research programs in classical and quantum relativistic dynamics of particles and fields."

founded

1998

Houston

Horwitz

Land

Gill

Fanchi

Schieve

2000 Tel Aviv

2002 Washington, D.C.

2004 Saas Fee

2006 Univ. of Connecticut

2008 Thessaloniki

2010 Haulien, Taiwan

2012 Florence

2014 Univ. of Connecticut

2016 Ljubljana

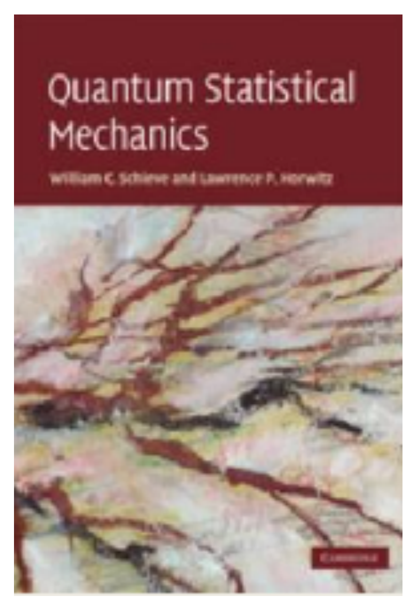
2018 Mérida

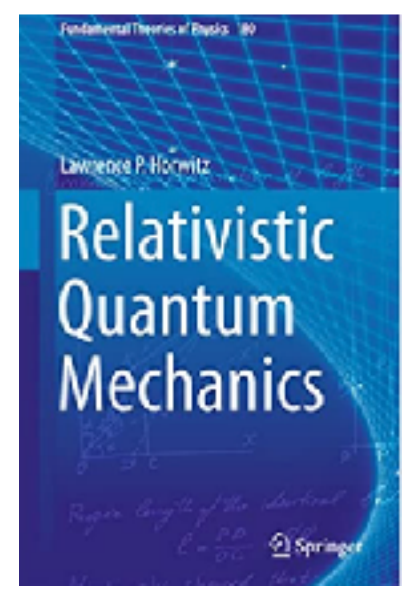
2020 Prague*

*in progress

Foundations of Physics

Alwyn Van de Merwe





IARD 2018

“The Era of Patience”

Rel. Field Theory

Quantum

Rel. Statistical Mechanics

Classical **Relativistic Dynamics**

Rel. Complex Systems

Covariant mechanics

Dynamical Time

Rel. Many-body Systems

Fundamentals of Relativity

Gravitation

Astrophysical

Applied Relativity

Neutron Stars

Galactic Dynamics

Special General

Quasars

Electrodynamics

…and any other unsolved or interesting problems in physics we felt like talking about

The Darwin Lagrangian

$$L_{\mathrm{D}} = \frac{q_1 q_2}{r} \frac{1}{2c^2} \mathbf{v}_1 \cdot [\mathbf{1} + \hat{\mathbf{r}}\hat{\mathbf{r}}] \cdot \mathbf{v}_2$$

Liénard-Wiechert Potentials

$$\varphi(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{(1-\mathbf{n}_s\cdot\boldsymbol{\beta}_s)|\mathbf{r}-\mathbf{r}_s|}\right)_{t_r}$$

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu_0 c}{4\pi}\left(\frac{q\boldsymbol{\beta}_s}{(1-\mathbf{n}_s\cdot\boldsymbol{\beta}_s)|\mathbf{r}-\mathbf{r}_s|}\right)_{t_r}$$

Radiative reaction

$$\mathbf{F}_{\mathrm{rad}} = \frac{2}{3}\frac{q^2}{c^3}\dot{\mathbf{a}}.$$

Higher order derivatives in equations of motion

Classical

Special Relativity

Many-Body Problem

Classical-Quantum Correspondence

?

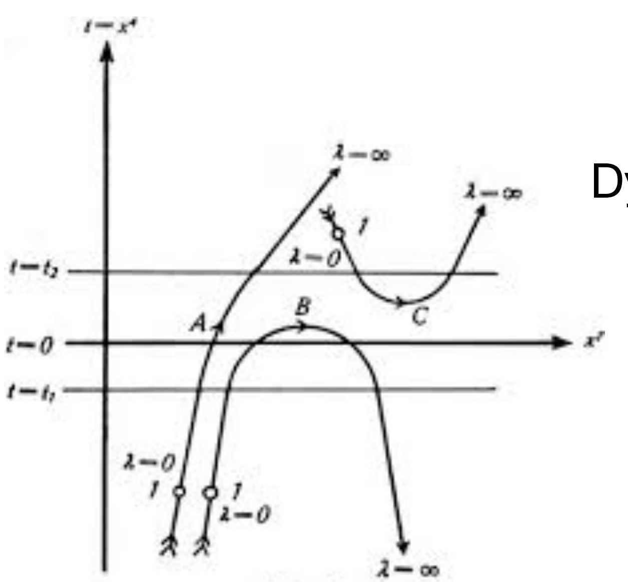


Fig. 1.

Dynamical Time

τ

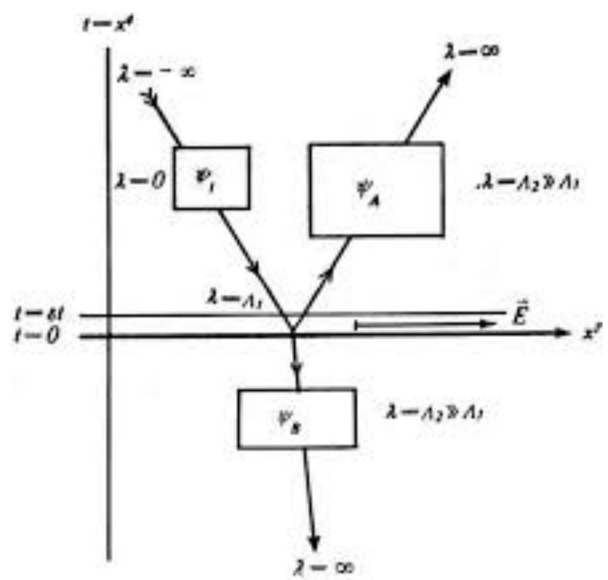
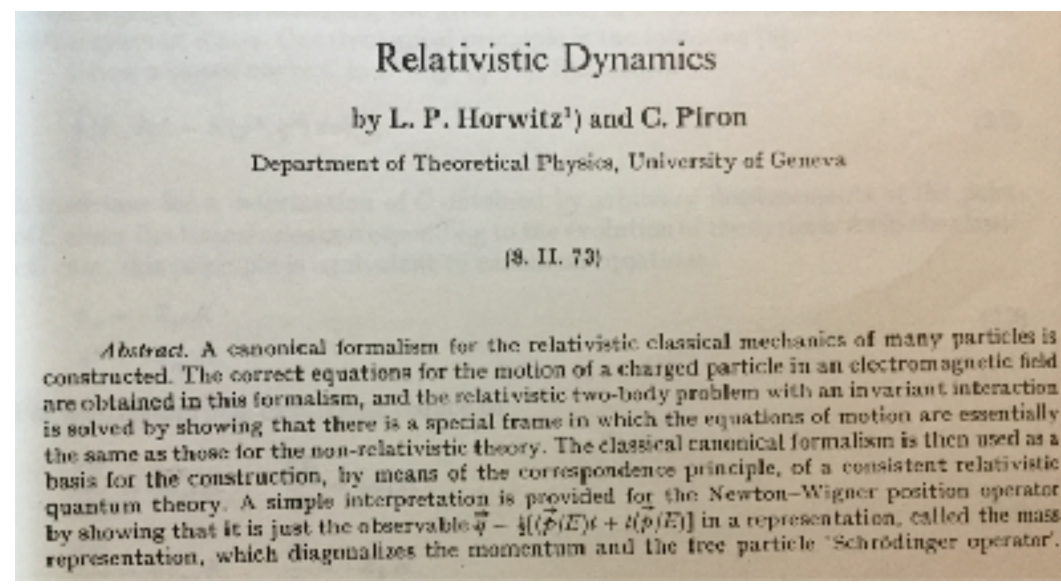


Fig. 2.

E.C.G. Stueckelberg, *Helvetica Physica Acta*, Vol. 14, 1941, pp. 51-80



Relativistic Dynamics

by L. P. Horwitz[1]) and C. Piron

Department of Theoretical Physics, University of Geneva

(8. II. 73)

Abstract. A canonical formalism for the relativistic classical mechanics of many particles is constructed. The correct equations for the motion of a charged particle in an electromagnetic field are obtained in this formalism, and the relativistic two-body problem with an invariant interaction is solved by showing that there is a special frame in which the equations of motion are essentially the same as those for the non-relativistic theory. The classical canonical formalism is then used as a basis for the construction, by means of the correspondence principle, of a consistent relativistic quantum theory. A simple interpretation is provided for the Newton–Wigner position operator by showing that it is just the observable $\vec{q} - \frac{1}{2}[(\vec{p}/E)t + t(\vec{p}/E)]$ in a representation, called the mass representation, which diagonalizes the momentum and the free particle 'Schrödinger operator'.

L.P. Horwitz and C. Piron, *Helvetica Physica Acta*, Vol. 46, 1973, pp. 316-326

Manifest Covariance

[graphics]

Perihelion Precession in the Special Relativistic Two-Body Problem

M. A. Trump and W. C. Schieve
Department of Physics and Ilya Prigogine Center for Studies in Statistical Mechanics and Complex Systems, The University of Texas at Austin, Austin, Texas, 78712
(May 21, 1998)

The classical two-body system with Lorentz-invariant Coulomb action-at-a-distance $V = -k/\rho$ is solved in 3+1 dimensions using the manifestly covariant Hamiltonian mechanics of Stückelberg. Particular solutions for the reduced motion are obtained which correspond to bound attractive, unbound attractive, and repulsive scattering motion. A lack of perihelion precession is found in the bound attractive orbit, and the semi-classical hydrogen spectrum subsequently contains no fine structure corrections. It is argued that this prediction is indicative of the correct classical special relativistic two-body theory.

The Synchronization Problem in Covariant Relativistic Dynamics

Matthew Trump[1] and W. C. Schieve[1]

Received November 9, 1996

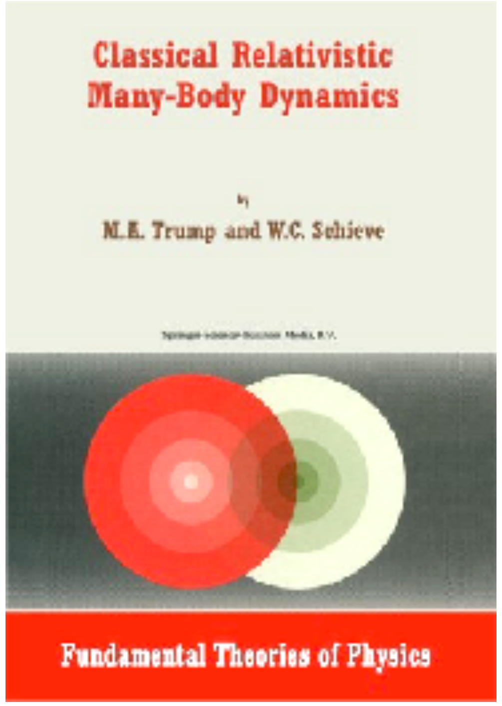
In the classical Stueckelberg–Horwitz–Piron relativistic Hamiltonian mechanics, a significant aspect of evolution of the classical n-body particle system with mutual interaction is the method by which events along distinct particle world lines are put into correspondence as a dynamical state. Approaches to this procedure are discussed in connection with active and passive symmetry principles.

Classical Scattering in the Covariant Two-Body Coulomb Potential

M. A. Trump[1] and W. C. Schieve[1]

Received November 5, 1997

The problem of two relativistically-moving pointlike particles of constant mass is undertaken in an arbitrary Lorentz frame using the classical Lagrangian mechanics of Stückelberg, Horwitz, and Piron. The particles are assumed to interact at events along their world lines at a common "world time," an invariant dynamical parameter which is not in general synchronous with the particle proper time. The Lorentz-scalar interaction is assumed to be the Coulomb potential (i.e., the inverse square spacetime potential) of the spacetime event separation. The classical orbit equations are found in 1 + 1 spacetime dimensions in the hyperbolic angle coordinates for the reduced problem. The solutions to the reduced motion in these coordinates are the spacetime generalizations of the nonrelativistic Kepler solutions, and they introduce an invariant eccentricity which is a function of other known constants of the motion for the reduced problem. Solutions compatible with physical scattering are obtained by the assumption that the eccentricity is a given function of the ratio of the particle masses.



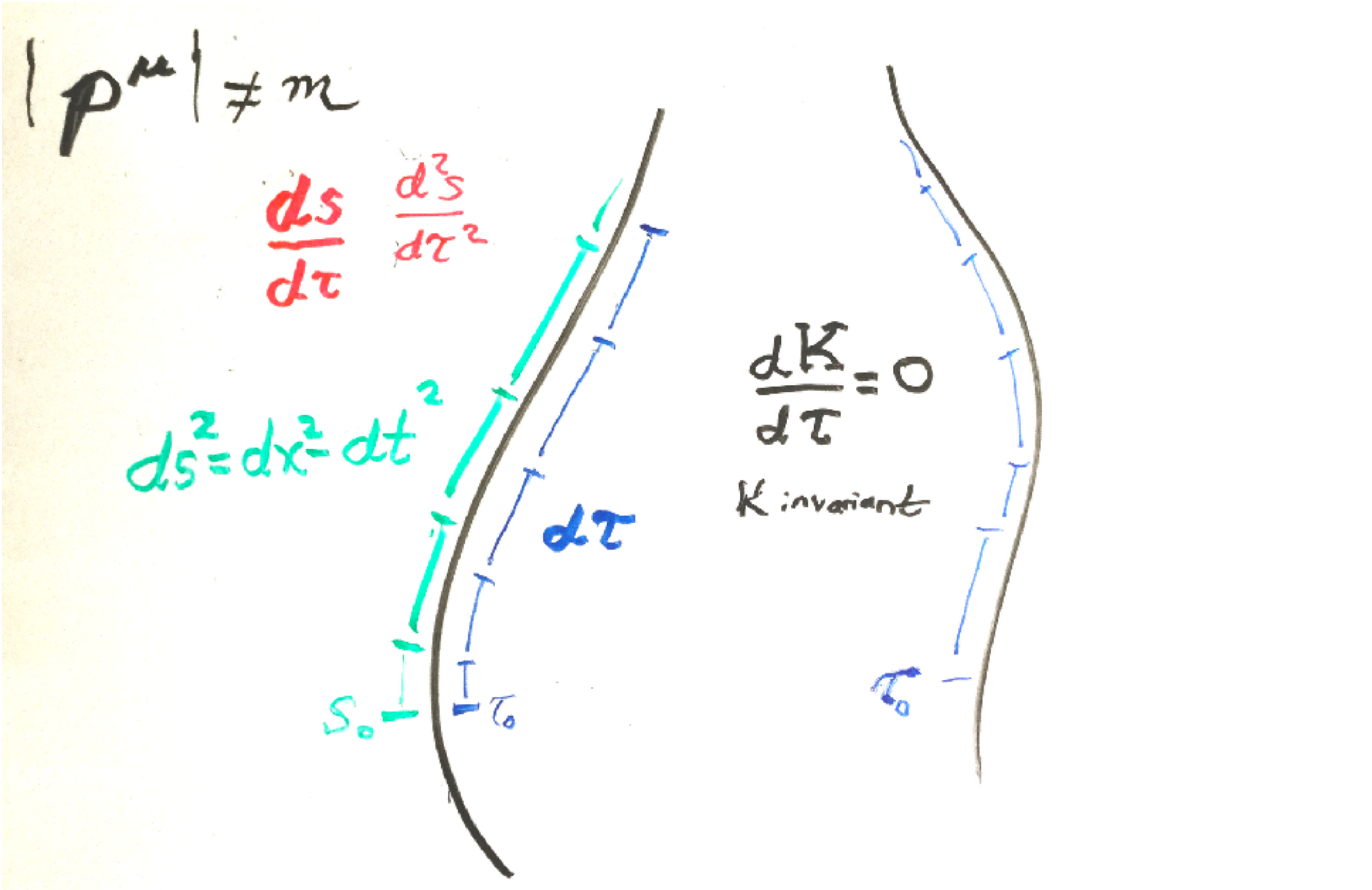
$$|p_i^\mu| = \pi_i(\tau) \neq m_i,$$

$$K = \sum_{i=1}^{2} \frac{p_i^\mu p_{\mu\, i}}{2m_i} + V(\rho),$$

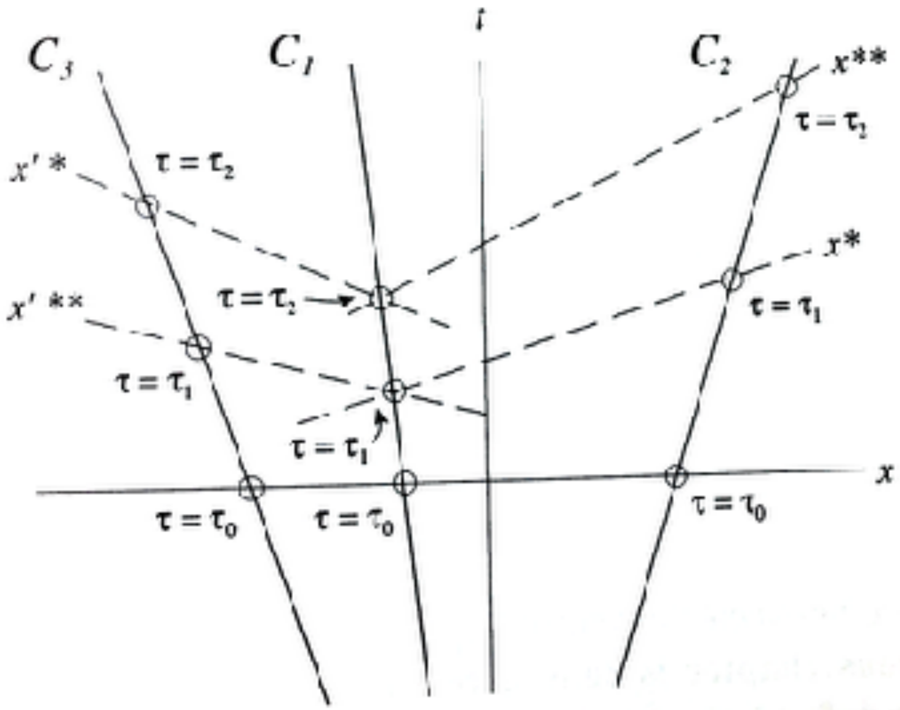
$$x_{i\,j}^{\mu}(\tau) = x_j^\mu(\tau) - x_i^\mu(\tau). \qquad \rho_{i\,j}(\tau) = \left| x_{i\,j}^{\mu} \right|,$$

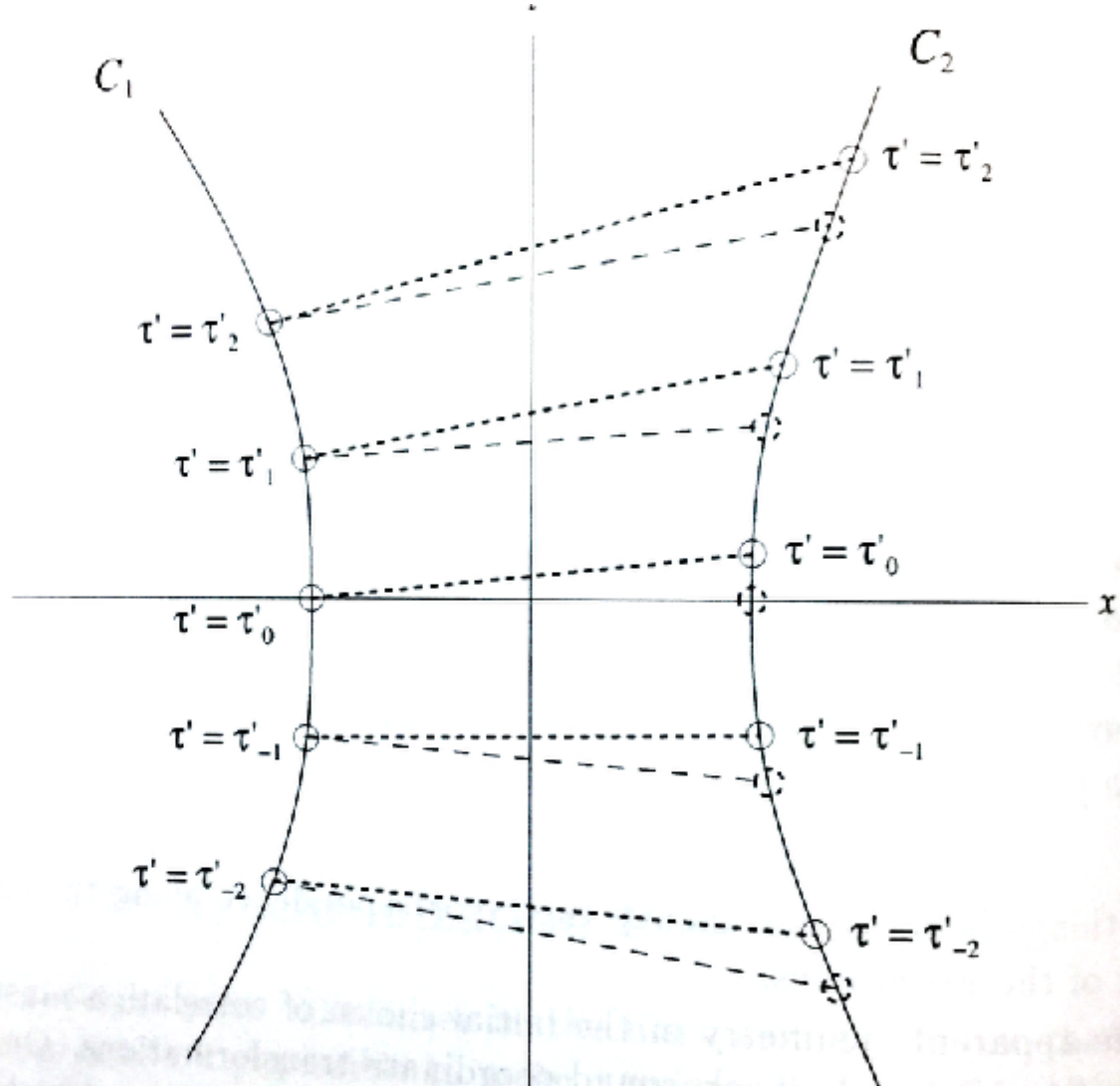
Off Mass Shell

Magnitude of 4-momentum is *not* constant

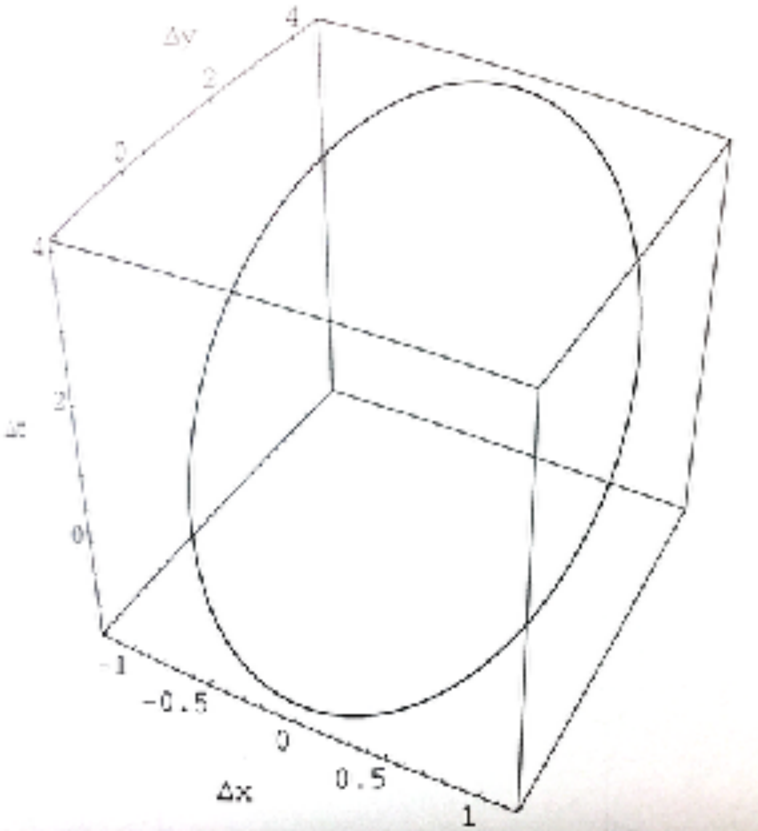


“The Synchronization Problem”





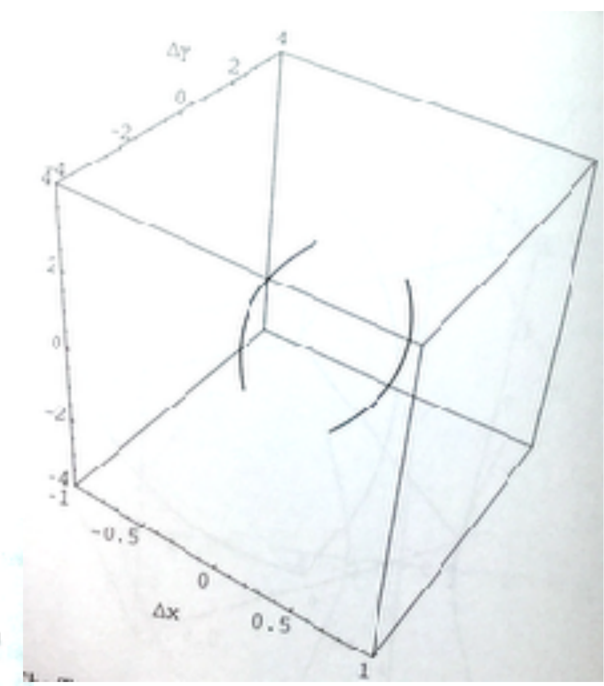
Two-Body Reduced Orbits

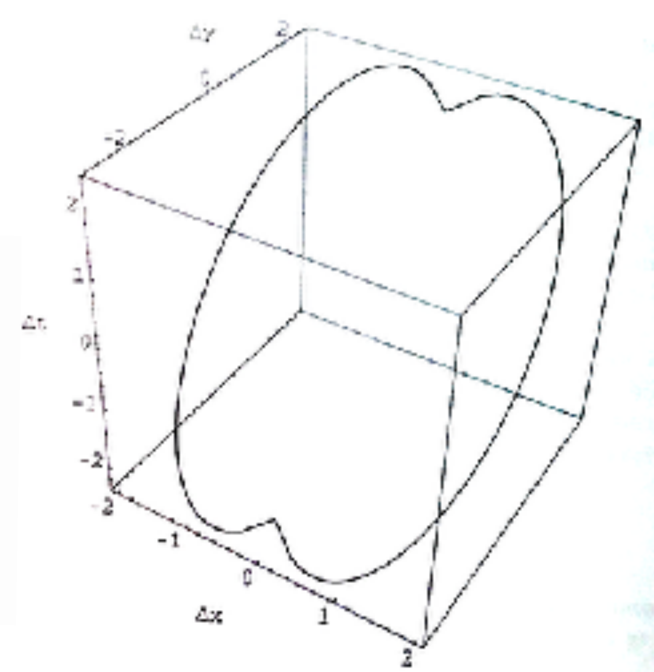


$$L = \frac{1}{2}\, m_i\, \dot{x}_1^{\mu}\, \dot{x}_{\mu\,1} + \frac{1}{2}\, m_i\, \dot{x}_2^{\mu}\, \dot{x}_{\mu\,2} + V(|x_2^{\mu} - x_1^{\mu}|).$$

$$X^{\mu}(\tau) = (\mathbf{X}(\tau),\, T(\tau))\,, \qquad x^{\mu}(\tau) = (\Delta\mathbf{x}(\tau),\, \Delta t(\tau))\,,$$

$$K = \sum_{i=1}^{2} \frac{p_i^{\mu}\, p_{\mu\,i}}{2m_i} + V(\rho),$$





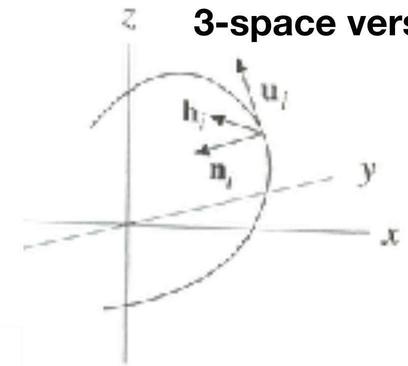
Type I: $\quad \dfrac{1}{\rho} = -\dfrac{mk}{\Lambda^2}\left(1 - e_1 \sinh\,(\beta - \beta_0)\right),$

Type II: $\quad \dfrac{1}{\rho} = -\dfrac{mk}{\Lambda^2}\left(1 - e_2 \cosh\,(\beta - \beta_0)\right),$

$$L_{\mathrm{D}} = \frac{q_1 q_2}{r} \frac{1}{2c^2} \mathbf{v}_1 \cdot [\mathbf{1} + \hat{\mathbf{r}}\hat{\mathbf{r}}] \cdot \mathbf{v}_2$$

The Covariant Serret-Frenet Equations

3-space version



Lorentz-invariant matrix elements

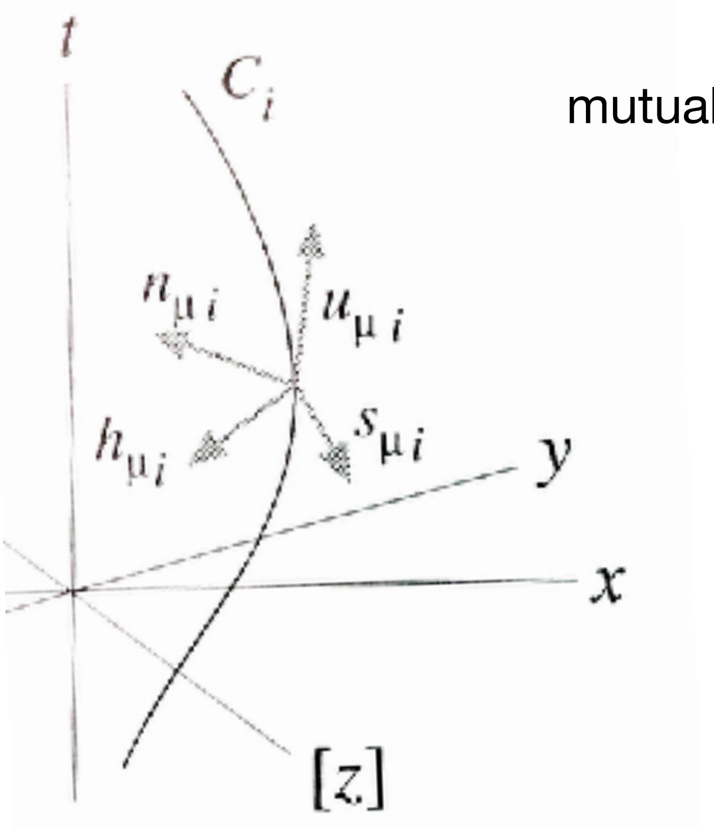
Completely determine world line as a function of ***arc-length***

$$\frac{d}{ds_i}\begin{bmatrix} u_i^\mu \\ n_i^\mu \\ h_i^\mu \\ s_i^\mu \end{bmatrix} = \begin{bmatrix} 0 & \xi_{1\,i} & 0 & 0 \\ \xi_{1\,i} & 0 & \xi_{2\,i} & 0 \\ 0 & -\xi_{2\,i} & 0 & \xi_{3\,i} \\ 0 & 0 & -\xi_{3\,i} & 0 \end{bmatrix}\begin{bmatrix} u_i^\mu \\ n_i^\mu \\ h_i^\mu \\ s_i^\mu \end{bmatrix},$$

$$\xi_{1\,i}(s_i) = \alpha_i,$$

$$\xi_{2\,i}(s_i) = \frac{\sigma_i}{\alpha_i},$$

$$\xi_{3\,i}(s_i) = \frac{\eta_i}{\sigma_i^2 \alpha_i},$$



mutually orthogonal unit 4-vectors

$$u_\mu u^\mu = 1$$

$$n_\mu = \frac{1}{\alpha} a_\mu$$

$$h_\mu = \frac{1}{\sigma}\left[\frac{da_\mu}{ds} - \alpha^2 u_\mu - \frac{\zeta^2}{\alpha^2} a_\mu\right]$$

$$s_\mu = \varepsilon_{\mu\nu\kappa\lambda} u^\nu n^\kappa h^\lambda$$

$$\alpha = (-a_\mu a^\mu)^{1/2}$$

$$\alpha = \left(a^2\gamma^4 + (\mathbf{v}\cdot\mathbf{a})^2\gamma^6\right)^{1/2}$$

$$\frac{dK}{d\tau} = 0. \quad ?$$

$$\zeta = \left(-\frac{da_\mu}{ds} a^\mu\right)^{1/2},$$

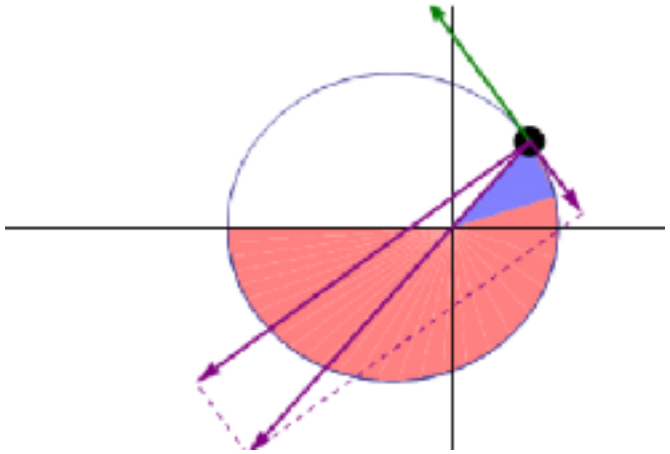
Svatá prostota!

$$\sigma = \frac{1}{\alpha} \cdot \left[b^2 a^2 \gamma^{10} - (\mathbf{a}\cdot\mathbf{b})^2 \gamma^{10} + b^2 (\mathbf{v}\cdot\mathbf{a})^2 \gamma^{12} + a^2 (\mathbf{v}\cdot\mathbf{b})^2 \gamma^{12} - 2(\mathbf{v}\cdot\mathbf{a})(\mathbf{v}\cdot\mathbf{b})(\mathbf{a}\cdot\mathbf{b})\gamma^{12}\right]^{1/2}$$

$$\eta = -\mathrm{d}\cdot(\mathrm{b}\times\mathrm{a})\,\gamma^{10}$$

$$u_i^\mu = \frac{dx_i^\mu}{ds_i}, \quad a_i^\mu = \frac{d^2 x_i^\mu}{ds_i^2}, \quad b_i^\mu = \frac{d^3 x_i^\mu}{ds_i^3}.$$

$$\mathbf{v}_i = \frac{d\mathbf{x}_i}{dt}, \quad \mathbf{a}_i = \frac{d^2\mathbf{x}_i}{dt^2}, \quad \mathbf{b}_i = \frac{d^3\mathbf{x}_i}{dt^3}, \quad \mathrm{d}_i(t) = \frac{d^4\mathbf{x}_i}{dt^4}$$



Děkuju